MAC 2233
Chapter 5 (Exam VI)
Practice For the Exam
Solutions (textbook)

use process of elimination to disqualify a, b, & d

so c

① $f(x) = (x-1)^{2/3} + 3 \quad [0,8]$

$f'(x) = \frac{2}{3}(x-1)^{-1/3}$

$f''(x) = -\frac{1}{3} \cdot \frac{2}{3}(x-1)^{-4/3}$

$= -\frac{2}{9}(x-1)^{-4/3}$

$\boxed{\text{stationary pts}}$

$f'(x) = \frac{2}{3(x-1)^{1/3}} = 0$

$2 \neq 0$ none

$\boxed{\text{singular points}}$

$f'(x) = \text{undefined}$

$x - 1 = 0$

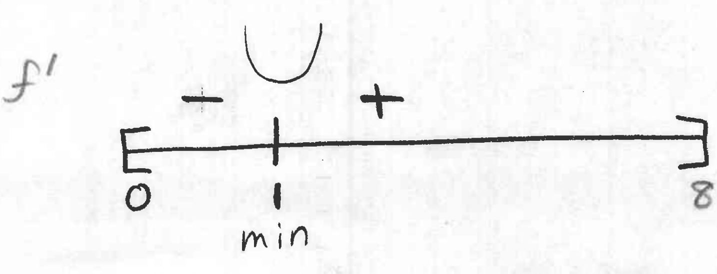
$x = 1$

check all possible relative extrema & endpoints

$f(1) = (1-1)^{2/3} + 3 = 3$

$f(0) = (0-1)^{2/3} + 3 = 4$

$f(8) = (8-1)^{2/3} + 3 = 3.65$



② $y = x^3 - 3x + 1$

$[-3, 3]$

mins ??

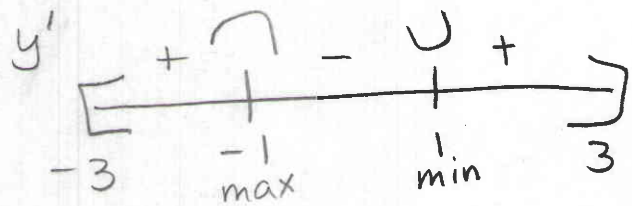
$y' = 3x^2 - 3$

$y' = 3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x+1)(x-1) = 0$

$x = -1$ & 1



$f'(-2) = 3(-2)^2 - 3 = (+)$

$f'(0) = 3(0)^2 - 3 = (-)$

$f'(2) = 3(2)^2 - 3 = (+)$

original

$f(-3) = (-3)^3 - 3(-3) + 1 = -17$ (min)

$f(-1) = 3$ (max)

$f(1) = -1$ (min)

$f(3) = 19$ (max)

A

③ include check of endpoints.

$(-3,2)$ (max) $\rightarrow$ local max

$(2,1)$ (min) $\rightarrow$ local min

since neither is the largest/smallest

$(-2,0)$ stationary pt (eventually an inflection pt)

---

④ $f(x) = \dfrac{e^{x^2-1}}{x}$ $x \neq 0$

f' is undefined at $x = 0$

$f'(x)$ quotient rule

f'

$+ \quad \frown \quad | \quad - \quad | \quad \text{infl.} \quad - \quad \smile \quad | \quad +$

$-\sqrt{\tfrac{1}{2}}$ max $\qquad 0 \qquad \sqrt{\tfrac{1}{2}}$ min

$f = e^{x^2-1}$ $\quad g = x$

$f' = 2xe^{x^2-1}$ $\quad g' = 1$

$$f'(x) = \frac{2xe^{x^2-1}(x) - (1)(e^{x^2-1})}{(x^2)}$$

$$f'(x) = \frac{e^{x^2-1}(2x^2-1)}{x^2}$$

$f(-\sqrt{\tfrac{1}{2}}) = -\sqrt{2}e^{-0.5}$

$f(\sqrt{\tfrac{1}{2}}) = \sqrt{2}e^{-0.5}$

$$f'(x) = 0 \qquad \frac{e^{x^2-1}(2x^2-1)}{x^2} = 0$$

$$2x^2 - 1 = 0$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \pm\sqrt{\frac{1}{2}}$$

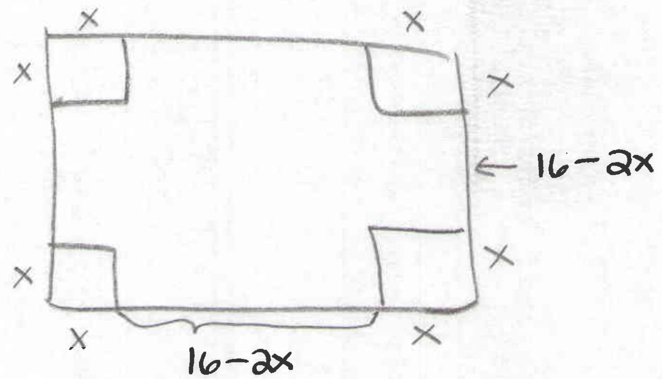
max

$(-\sqrt{\tfrac{1}{2}}, -\sqrt{2}e^{-0.5})$

min

$(\sqrt{\tfrac{1}{2}}, \sqrt{2}e^{-0.5})$

(5)



$V = l \cdot w \cdot h$

$= (16-2x)(16-2x)(x)$

$= x(256 - 32x - 3x + 4x^2)$

$= x(4x^2 - 64x + 256)$

$= 4x^3 - 64x^2 + 256x$

$V' = 12x^2 - 128x + 256$

$4(3x^2 - 32x + 64) = 0$

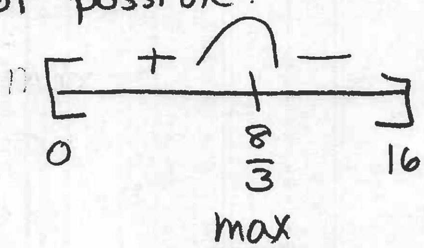
$4(3x - 8)(x - 8) = 0$

$x = \frac{8}{3} \qquad x = 8$

If $x = 0$, then $16 - 2x = 0$

So, not possible.

$\boxed{x = \frac{8}{3}}$



(6) $S(t) = -16t^2 + 480t$

$S'(t) = -32t + 480$

$-32t + 480 = 0$

$-32t = -480$

$t = 15$

$S(15) = -16(15)^2 + 480(15)$

$= 3600$

(C)

(7) d & f

(D)

(8) B

(9) $f(x) = -2x^2 + 2x - 1$

$f'(x) = -4x + 2$

$f''(x) = -4$

alway concave down

(D)

(10) $f(x) = x^3 - 3x^2 - 24x + 10$

$f'(x) = 3x^2 - 6x - 24$

$f''(x) = 6x - 6$

$f''(x) = 6x - 6 = 0$

$6x = 6$

$x = 1$

$f(1) = (1)^3 - 3(1)^2 - 24(1) + 10$

$= 1 - 3 - 24 + 10$

$= 11 - 27 = -16$

(D)

(11) $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

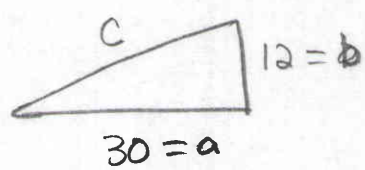
$$\frac{dr}{dt} = 4 \text{cm/s}$$

$r = 2\text{cm}$

(A)

$$\frac{dA}{dt} = 2\pi(2)(4) = \boxed{50.27 \text{ cm}^2/\text{s}}$$

(12)



6'' per sec.

$$a^2 + b^2 = c^2$$
$$(30)^2 + (12)^2 = c^2$$
$$900 + 144 = c^2$$
$$32.311 = c$$

$\frac{da}{dt} = 6$

$\frac{db}{dt} = 0$

$\frac{dc}{dt} =$ what we need to find

$$a^2 + b^2 = c^2$$

$$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(30)(6) + 2(12)(0) = 2(32.311)\frac{dc}{dt}$$

$$360 = 64.622\frac{dc}{dt}$$

$$\boxed{5.57 \text{ in/sec} = \frac{dc}{dt}}$$

(13) $E = 0.5$

(D)

(14) $q = 80 - 2p \qquad 0 \le p \le 40$

$$\frac{dq}{dp} = -2$$

$$E = -\frac{dq}{dp} \cdot \frac{p}{q} = \frac{-(-2) \cdot p}{80 - 2p}$$

$$= \frac{2p}{80-2p} = \boxed{\frac{p}{40-p}}$$

(B)

(15) $x = 100\sqrt{225-p} \quad 0 \le p \le 225$

max price when $E = 1$

$E = -\dfrac{dx}{dp} \cdot \dfrac{p}{x}$ $\quad \dfrac{dx}{dp} = 100 \cdot \frac{1}{2}(225-p)^{-1/2}(-1)$

$= \dfrac{-50}{\sqrt{225-p}}$

$E = \dfrac{50}{\sqrt{225-p}} \cdot \dfrac{p}{100\sqrt{225-p}} = \dfrac{p}{2(225-p)}$

$E = \dfrac{p}{2(225-p)} = 1$

$p = 2(225-p)$

$p = 450 - 2p$

$3p = 450$

$\boxed{p = \$150}$

(A)

(16) $q = 3000 - 750p$ $\quad$ inelastic demand

$0.75 \le p \le 2.50$ $\quad 0 < E < 1$

$E = -\dfrac{dq}{dp} \cdot \dfrac{p}{q} = \dfrac{-(-750) \cdot p}{3000-750p} = \dfrac{750p}{3000-750p}$

$p < \dfrac{3000}{1500}$

$\dfrac{dq}{dp} = -750$

$\dfrac{750p}{3000-750p} < 1$

$p < 2$ (max)

(C)

$750p < 3000 - 750p$

$1500p < 3000$

so $\$0.75 < p < \$2.00$

17 $q = 1000 - 20p$

$$E = -\frac{dq}{dp} \cdot \frac{p}{q} = \frac{-(-20) \cdot p}{1000 - 20p} = \frac{20p}{1000 - 20p}$$

$$E = \frac{p}{50 - p}$$

$$\boxed{E(22)} = \frac{22}{50 - 22} = \boxed{0.7857}$$

max when $E = 1$

$$E = \frac{p}{50 - p} = 1$$

$$p = 50 - p$$

$$2p = 50$$

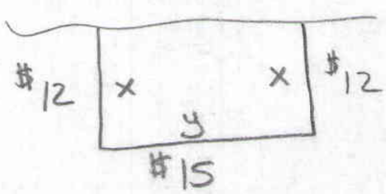
$$\boxed{p = \$25}$$

$$R = p \cdot q$$

$$= (25)(1000 - 20(25))$$

$$= \boxed{\$12{,}500 \text{ max revenue}}$$

Bonus 1



$3600 available.

$$A = xy$$

$$12x + 15y + 12x = 3600$$

$$24x + 15y = 3600$$

$$15y = 3600 - 24x$$

$$y = \frac{3600 - 24x}{15} = \frac{1200 - 8x}{5}$$

$$A = xy = x\left(\frac{1200 - 8x}{5}\right)$$

$$= \frac{1200}{5}x - \frac{8}{5}x^2$$

$$= 240x - \frac{8}{5}x^2$$

$$A' = 240 - \frac{16}{5}x = 0$$

$$240 = +\frac{16}{5}x$$

$$75 = x$$

max area

$$A(75) = 240(75) - \frac{8}{5}(75)^2$$

$$\boxed{9000 \text{ sq ft}}$$

Bonus 2

$$F(x) = \frac{2600}{1+x}$$

$$\frac{dx}{dt} = 2 \text{ ppm/yr}$$

$$F = 600$$

$$\Rightarrow \quad 600 = \frac{2600}{1+x}$$

$$1+x = \frac{2600}{600}$$

$$\boxed{x = 3\tfrac{1}{3}}$$

$$\frac{dF}{dt} = \frac{d}{dt}\left(\frac{2600}{1+x}\right)$$

quotient rule

$f = 2600 \qquad g = 1+x$

$f' = 0 \qquad g' = \frac{dx}{dt}$

$$\frac{dF}{dt} = \frac{0(1+x) - (2600)\left(\frac{dx}{dt}\right)}{(1+x)^2}$$

$$\frac{dF}{dt} = \frac{-2600\,\frac{dx}{dt}}{(1+x)^2}$$

$$\frac{dF}{dt} = \frac{-2600(2)}{(1+3\frac{1}{3})^2} = \boxed{-276.9 \text{ fish/year}}$$

$$4\tfrac{1}{3} = \frac{13}{3}$$

D